

# Integral Graphs

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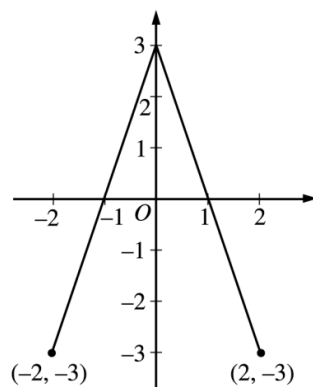
## Question 1

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Derivative Graphs, Integration - Area Under A Curve, Increasing/Decreasing, Concavity, Integration Graphs

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 4



Graph of  $f$

4. The graph of the function  $f$  shown above consists of two line segments. Let  $g$  be the function given by  $g(x) = \int_0^x f(t) dt$ .
- (a) Find  $g(-1)$ ,  $g'(-1)$ , and  $g''(-1)$ .
  - (b) For what values of  $x$  in the open interval  $(-2, 2)$  is  $g$  increasing? Explain your reasoning.
  - (c) For what values of  $x$  in the open interval  $(-2, 2)$  is the graph of  $g$  concave down? Explain your reasoning.
  - (d) On the axes provided, sketch the graph of  $g$  on the closed interval  $[-2, 2]$ .  
(Note: The axes are provided in the pink test booklet only.)

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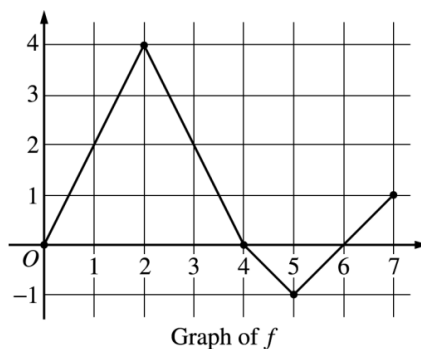
## Question 2

Qualification: AP Calculus AB

Areas: Integration, Differentiation, Applications of Differentiation

Subtopics: Derivative Graphs, Rates of Change (Average), Mean Value Theorem, Points Of Inflection, Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second), Integration Graphs

Paper: Part B-Non-Calc / Series: 2003-Form-B / Difficulty: Medium / Question Number: 5



5. Let  $f$  be a function defined on the closed interval  $[0, 7]$ . The graph of  $f$ , consisting of four line segments, is shown above. Let  $g$  be the function given by  $g(x) = \int_2^x f(t) dt$ .
- (a) Find  $g(3)$ ,  $g'(3)$ , and  $g''(3)$ .
  - (b) Find the average rate of change of  $g$  on the interval  $0 \leq x \leq 3$ .
  - (c) For how many values  $c$ , where  $0 < c < 3$ , is  $g'(c)$  equal to the average rate found in part (b)? Explain your reasoning.
  - (d) Find the  $x$ -coordinate of each point of inflection of the graph of  $g$  on the interval  $0 < x < 7$ . Justify your answer.

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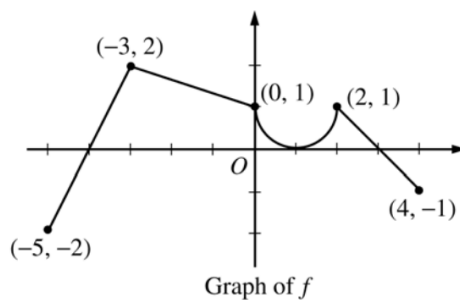
### Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 5



5. The graph of the function  $f$  shown above consists of a semicircle and three line segments. Let  $g$  be the function given by  $g(x) = \int_{-3}^x f(t) dt$ .
- (a) Find  $g(0)$  and  $g'(0)$ .
  - (b) Find all values of  $x$  in the open interval  $(-5, 4)$  at which  $g$  attains a relative maximum. Justify your answer.
  - (c) Find the absolute minimum value of  $g$  on the closed interval  $[-5, 4]$ . Justify your answer.
  - (d) Find all values of  $x$  in the open interval  $(-5, 4)$  at which the graph of  $g$  has a point of inflection.

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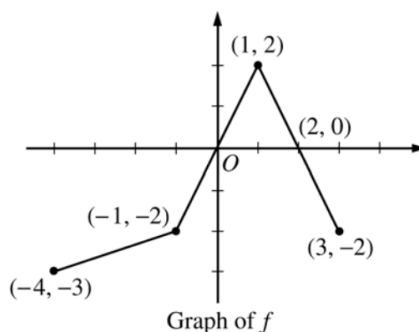
## Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Fundamental Theorem of Calculus (Second), Points Of Inflection, Increasing/Decreasing, Integration Technique – Geometric Areas, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2005-Form-B / Difficulty: Somewhat Challenging / Question Number: 4



4. The graph of the function  $f$  above consists of three line segments.

- (a) Let  $g$  be the function given by  $g(x) = \int_{-4}^x f(t) dt$ . For each of  $g(-1)$ ,  $g'(-1)$ , and  $g''(-1)$ , find the value or state that it does not exist.
- (b) For the function  $g$  defined in part (a), find the  $x$ -coordinate of each point of inflection of the graph of  $g$  on the open interval  $-4 < x < 3$ . Explain your reasoning.
- (c) Let  $h$  be the function given by  $h(x) = \int_x^3 f(t) dt$ . Find all values of  $x$  in the closed interval  $-4 \leq x \leq 3$  for which  $h(x) = 0$ .
- (d) For the function  $h$  defined in part (c), find all intervals on which  $h$  is decreasing. Explain your reasoning.

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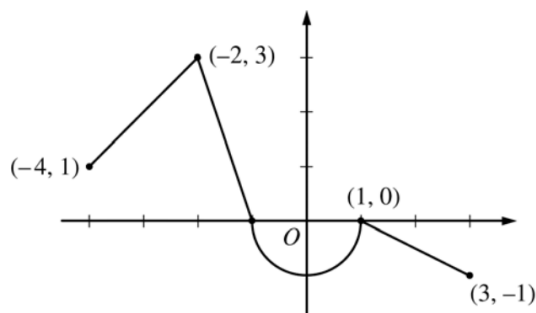
## Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second), Local or Relative Minima and Maxima, Points Of Inflection, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 3



Graph of  $f$

3. Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .
- (a) Find the values of  $g(2)$  and  $g(-2)$ .
  - (b) For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.
  - (c) Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
  - (d) For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.
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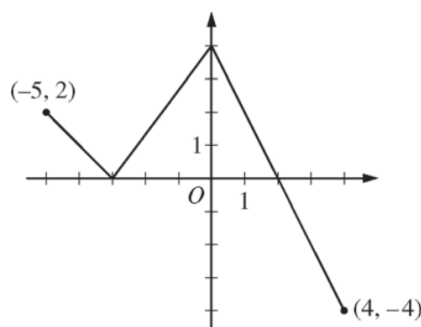
## Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Increasing/Decreasing, Concavity, Differentiation Technique - Quotient Rule, Tangents To Curves, Differentiation Technique – Chain Rule, Integration Graphs

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 3



Graph of  $f$

3. The function  $f$  is defined on the closed interval  $[-5, 4]$ . The graph of  $f$  consists of three line segments and is shown in the figure above. Let  $g$  be the function defined by  $g(x) = \int_{-3}^x f(t) dt$ .
- (a) Find  $g(3)$ .
- (b) On what open intervals contained in  $-5 < x < 4$  is the graph of  $g$  both increasing and concave down? Give a reason for your answer.
- (c) The function  $h$  is defined by  $h(x) = \frac{g(x)}{5x}$ . Find  $h'(3)$ .
- (d) The function  $p$  is defined by  $p(x) = f(x^2 - x)$ . Find the slope of the line tangent to the graph of  $p$  at the point where  $x = -1$ .
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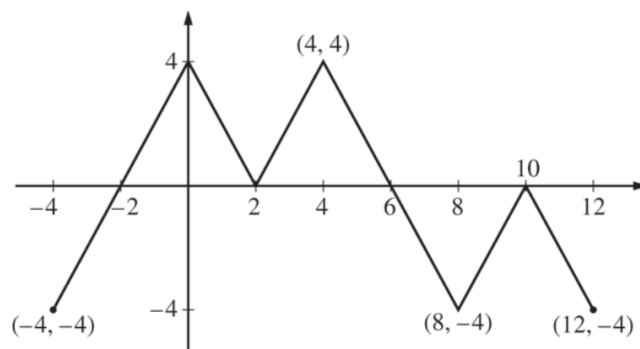
## Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Points Of Inflection, Global or Absolute Minima and Maxima, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 3



Graph of  $f$

3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined

$$\text{by } g(x) = \int_2^x f(t) \, dt.$$

- (a) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 10$ ? Justify your answer.
- (b) Does the graph of  $g$  have a point of inflection at  $x = 4$ ? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of  $g$  on the interval  $-4 \leq x \leq 12$ . Justify your answers.
- (d) For  $-4 \leq x \leq 12$ , find all intervals for which  $g(x) \leq 0$ .

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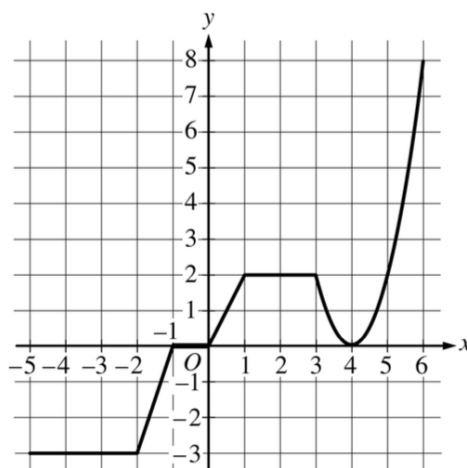
## Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing, Concavity, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Medium / Question Number: 3



Graph of  $g$

3. The graph of the continuous function  $g$ , the derivative of the function  $f$ , is shown above. The function  $g$  is piecewise linear for  $-5 \leq x < 3$ , and  $g(x) = 2(x - 4)^2$  for  $3 \leq x \leq 6$ .
- If  $f(1) = 3$ , what is the value of  $f(-5)$ ?
  - Evaluate  $\int_1^6 g(x) \, dx$ .
  - For  $-5 < x < 6$ , on what open intervals, if any, is the graph of  $f$  both increasing and concave up? Give a reason for your answer.
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $f$ . Give a reason for your answer.

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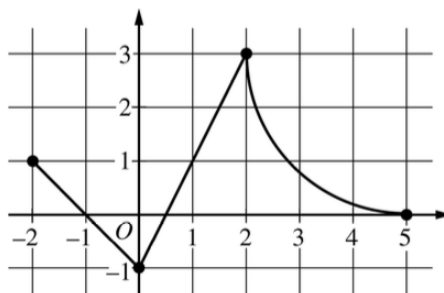
## Question 9

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First), Global or Absolute Minima and Maxima, Calculating Limits Algebraically, Integration Graphs

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 3



Graph of  $f$

3. The continuous function  $f$  is defined on the closed interval  $-6 \leq x \leq 5$ . The figure above shows a portion of the graph of  $f$ , consisting of two line segments and a quarter of a circle centered at the point  $(5, 3)$ . It is known that the point  $(3, 3 - \sqrt{5})$  is on the graph of  $f$ .

(a) If  $\int_{-6}^5 f(x) \, dx = 7$ , find the value of  $\int_{-6}^{-2} f(x) \, dx$ . Show the work that leads to your answer.

(b) Evaluate  $\int_3^5 (2f'(x) + 4) \, dx$ .

(c) The function  $g$  is given by  $g(x) = \int_{-2}^x f(t) \, dt$ . Find the absolute maximum value of  $g$  on the interval  $-2 \leq x \leq 5$ . Justify your answer.

(d) Find  $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$ .

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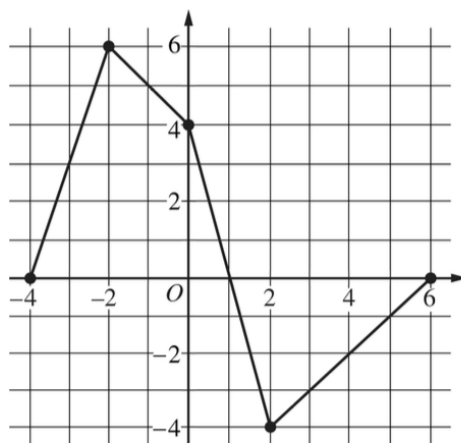
## Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Concavity, Fundamental Theorem of Calculus (Second), Differentiation Technique – Product Rule, L'Hôpital's Rule, Mean Value Theorem, Rates of Change (Average), Integration Technique – Geometric Areas, Integration Graphs

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of  $f$

4. Let  $f$  be a continuous function defined on the closed interval  $-4 \leq x \leq 6$ . The graph of  $f$ , consisting of four line segments, is shown above. Let  $G$  be the function defined by  $G(x) = \int_0^x f(t) \, dt$ .
- (a) On what open intervals is the graph of  $G$  concave up? Give a reason for your answer.
- (b) Let  $P$  be the function defined by  $P(x) = G(x) \cdot f(x)$ . Find  $P'(3)$ .
- (c) Find  $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$ .
- (d) Find the average rate of change of  $G$  on the interval  $[-4, 2]$ . Does the Mean Value Theorem guarantee a value  $c$ ,  $-4 < c < 2$ , for which  $G'(c)$  is equal to this average rate of change? Justify your answer.

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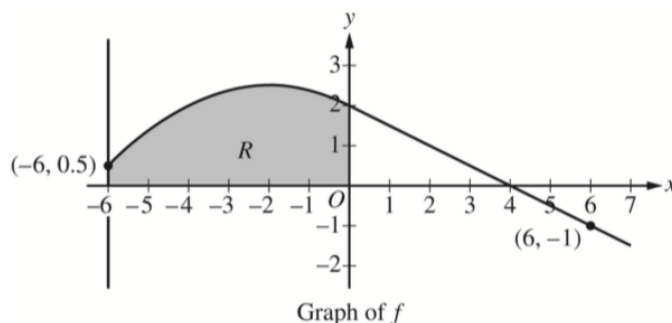
## Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 4



4. The graph of the differentiable function  $f$ , shown for  $-6 \leq x \leq 7$ , has a horizontal tangent at  $x = -2$  and is linear for  $0 \leq x \leq 7$ . Let  $R$  be the region in the second quadrant bounded by the graph of  $f$ , the vertical line  $x = -6$ , and the  $x$ - and  $y$ -axes. Region  $R$  has area 12.
- (a) The function  $g$  is defined by  $g(x) = \int_0^x f(t) \, dt$ . Find the values of  $g(-6)$ ,  $g(4)$ , and  $g(6)$ .
- (b) For the function  $g$  defined in part (a), find all values of  $x$  in the interval  $0 \leq x \leq 6$  at which the graph of  $g$  has a critical point. Give a reason for your answer.
- (c) The function  $h$  is defined by  $h(x) = \int_{-6}^x f'(t) \, dt$ . Find the values of  $h(6)$ ,  $h'(6)$ , and  $h''(6)$ . Show the work that leads to your answers.

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